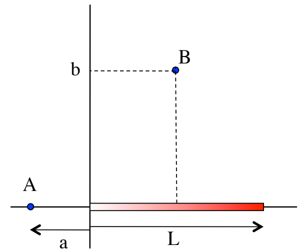
Problem 25.42

An insulating rod whose charge density is

$$\lambda = \alpha x,$$

where α is a positive constant.



a.) What are the units of α?

λ's units are *"coulomb's per meter."* What do you have to multiply *meters* by to get that?

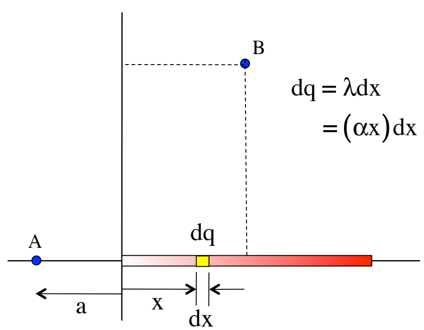
"coulomb's per meter-squared"

b.) Determine the electrical potential at Point A (I'm calling the distance to that point "a"—using "d" could get confusing if there are differentials in the mix).

Again, we need to start by determining the electrical potential at A due to a differential bit of charge dq located at an arbitrary point "x" along the axis (see the sketch on the next page).

b.)

$$dV = k\frac{dq}{(x+a)} = k\frac{\lambda dx}{(x+a)} = k\frac{(\alpha x)dx}{(x+a)}$$



$$dq = \lambda dx = (\alpha x)dx$$

The integral we will end up with is going to be somewhat complex (hence the need to use the entire next page to present it). When you run into something like this, you can attempt to grunt through it using the Calculus you have learned, or you can go to a book of integrals, find the appropriate form and evaluate that. I mention this because that is exactly what I have done here. (Yes, I know, I'm a veritable intellectual slug!)

$$\Rightarrow \quad V = \int dV = k\int \frac{(\alpha x)dx}{(x+a)}$$

$$= k\alpha \int_{x=0}^{L} \frac{x}{(x+a)}dx$$

$$= k\alpha\left[(x+a) - a\ln(x+a)\right]\Big|_{x=0}^{L}$$

$$= k\alpha\left[\left[(L+a) - a\ln(L+a)\right] - \left[(a) - a\ln(a)\right]\right]$$

$$= k\alpha\left[L + a - a - \left(a\ln(L+a) - a\ln(a)\right)\right]$$

$$= k\alpha\left[L - a\ln\left(\frac{L+a}{a}\right)\right]$$

$$= k\alpha\left[L - a\ln\left(\frac{L}{a}+1\right)\right]$$